INTERACTION OF A TORNADO-LIKE VORTEX WITH SOLID BOUNDARIES

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Interest in the study of concentrated columnar vortices has been largely motivated by efforts to determine the structure, mechanism of formation, and energy source of such natural phenomena as tornadoes [1, 2] and "dust devils" [1, 3-5]. tornado-like vortices are produced experimentally by various techniques (e.g., with the presence of a sink in a rotating fluid [6-10]). In this setting, the problem of the energy sources of real vortices is sidestepped, and only the flow structure is investigated. Vortices have been generated [11-13] by a mechanism that is possibly closer to reality. The source of vorticity concentration in this case is an unstable stratification of a water—air mixture [11] or air heated from below [12, 13]. Theoretical models based on unstable stratification have been devised [12, 14], in one case with the assumption that only a thin layer near the bottom surface [12] and, in the other, that the entire atmosphere [14] is unstable. Qualitatively different results are obtained in either case. For example, if the vortex radius is taken to be the distance at which the rotational velocity attains a maximum, then it is inferred from [12] that its value increases with height z along the vortex, whereas [14] implies that it does not depend on z.

1. The apparatus used to generate and investigate tornado-like vortices comprises a cylindrical vessel of heat-resistant glass. The cylinder has a height of 130 mm and a diamete of 100 mm. It is mounted vertically, filled with water, and heated from below. The liquid is set in motion by a rotating transparent disk mounted near the top of the open part of the cylinder. The disk has a diameter of 90 mm. The experimentally generated flow has axial symmetry. We introduce a coordinate system (r, φ, z), where r is the radius and φ is the azimuth angle. The z axis coincides with the flow symmetry axis. We denote by u, v, w the velocity components corresponding to the coordinates r, φ, z. The flow velocities are measured by the hydrogen-bubble method [15]. Copper wires with a diameter of 50 μm are used for this purpose. The following functions are determined: v(r) and w(r) for fixed values of z; v(z) for fixed values of r. The functions v(r) and w(r) are determined from photographs of the line of bubbles generated by a horizontal wire. The wire is stretched across the diameter of the cylinder at heights of 1 and 3 cm above the bottom. In the first case the bubbles are photographed from topside at an angle of 10° relative to the z axis, and in the second case from the side along the normal to the wire. The function v(z) is determined by photographing the bubbles from a vertical wire. The latter is set at a distance of 20 and 30 mm from the center of the cylinder. The bubbles are photographed from the side along the radius through the wire. In every case the bubble line is photographed on film with a certain delay time τ after bubble formation. The photographs are taken for different values of the disk rotation frequency f and the water depth l in the cylinder.

In the absence of heating, the liquid outside thin boundary layers at the solid surfaces rotates almost as a rigid body. The frequency of rotation is roughly equal to 0.2f. Maxworthy [9] has obtained a similar result. With heating at the center of the cylinder, a vertical vortex is formed. Figure 1 gives a photograph of this vortex, which is visualized by the addition of dye in the lower boundary layer (f = 0.9 rps), Fig. 2 represents a typical photograph of the bubbles from a vertical wire (τ = 0.53 sec, f = 0.9 rps), and Fig. 3 represents topside photographs of the bubble line from a horizontal wire. The upper frame corresponds to f = 0.9 rps, and the lower frame to f = 1.54 rps. In both cases l = 12 cm and τ = 0.11 sec. Figure 4 represents photographs of a bubble line, illustrating the function w(r) at a height of 1 cm (τ = 0.53 sec, f = 0.9 rps).

Making use of the experimental data, we note some important qualitative features of the flow. Immediately after exit from the boundary layer, a vortex explosion takes place [9] (Fig. 1). We infer from the data of Fig. 2 that in the first approximation v does not de-

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Fig. 1

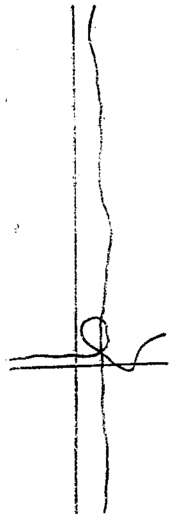

Fig. 2

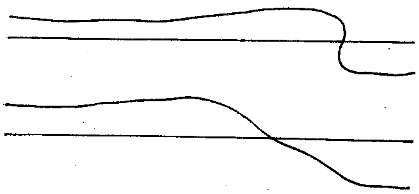

Fig. 3

Fig. 4

pend on the height z. According To Fig. 3, it is possible within the vortex to discern a core, in which the rotation is close to that of a rigid body, while outside the core it is nearly potential. Figure 3 and Table 1 illustrate the effect observed in the experiment. With an increase in f($f \geqslant 0.9$ rps, $l = 12$ cm) or with a decrease in l($f = 0.9$ rps, $l \leqslant 12$ cm) the radius of the core increases sharply, but with an increase in f($f \leqslant 0.9$ rps, $l = 12$ cm), it remains practically unchanged. In Table 1, r_0 is the radius of the core, and A_∞ is the value of vr at $r = r_0$. According to Fig. 4, the vertical-velocity profile is close to a step function. The radius r_* of the updraft zone is approximately equal to the radius of the core. The flow inside the core is directed upward.

Along with the determination of the velocity field, we have measured the temperature as well. A mercury thermometer is used for this purpose. The measurements indicate that the temperature in the core is 1-3°C higher than outside the core. In the region exterior to the core we observe roughly a 1°C increase in the temperature as the thermometer is lowered from a height of 10 cm to 1 cm. Also, the temperature of the water in the cylinder increases approximately at the rate of 1°C every 30 sec. Measurements in the core and at various heights show that the characteristic temperature is 1°C. The characteristic time of the process, in turn, reckoned according to the frequency of rotation of the core or the ascension time of the liquid in the core, amounts to a few seconds. Thus, the variation of the water temperature during the characteristic time of the process is small in comparison with the characteristic temperature. We note that in all the experiments the flow was laminar and the heating conditions were constant.

2. On the basis of the experimental data we construct a theoretical model of the observed effect. The principal objective here is to obtain expressions for the radius of the core and to determine its compaction effect. A schematic diagram of the flow field in the cylinder in an axial cross section is given in Fig. 5. Region I is the boundary layer,

TABLE 1

f, rps	0,9	1,2	1,54	0,9
l, cm	12	12	12	8,5
A_∞, cm^2/sec	1,3	2,55	5,3	1,9
r_0, cm	0,3	0,55	1,1	0,55
$A_\infty/r_0 l$, sec^{-1}	0,36	0,39	0,4	0,4

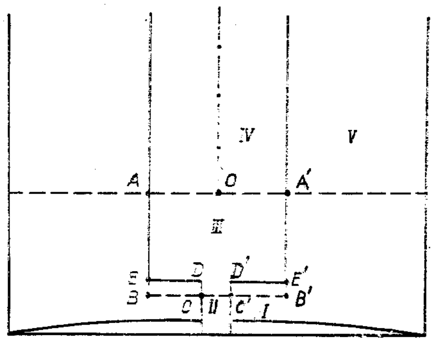

Fig. 5

in region II the flow from the boundary layer emerges upward in the form of a narrow twisted stream, vortex explosion takes place in region III, the vortex core is formed in region IV, and in region V the flow is close to potential. Let us examine regions IV and V. We place the origin at point O. We state the following main assumptions:

a) The flow is laminar, steady, and cylindrically symmetric.

b) The Boussinesq approximation holds.

c) The azimuthal component of the velocity is independent of z in the first approximation, so that

$$vr = A(r) + \varepsilon A_1(r, z), \tag{2.1}$$

where $\varepsilon \ll$ and $A \sim A_1$.

d) Since $r_0 \approx r_*$, we assume that $r_0 = r_*$.

e) The region V (Fig. 5) extends to infinity in the radial direction.

f) In each cross section z = const the axial velocity profile w(r) and the quantity $\Theta(r) = T - T_*$, where T is the temperature and T_* is the temperature at an infinite distance from the vortex axis, can be approximated by step functions in r:

$$w(r) = \begin{cases} W(z), \\ 0, \end{cases} \quad \theta(r) = \begin{cases} \Theta(z), & 0 \leqslant r \leqslant r_0, \\ 0, & r > r_0 \end{cases} \tag{2.2}$$

(we note that if in the approximation of θ we replace r_0 by $r_1 \neq r_0$, the qualitative results remain unchanged, except that certain coefficients in the equations acquire factors of the type r_1^2/r_0^2 of order unity).

g) Inasmuch as $r_0 \ll l$ and the variations of the velocity with height and along the radius of the core are of the same order, we invoke the approximation of an axial boundary layer.

We now derive the system of equations. We use assumptions a), b), e), g), an equation of state in the form $\rho = \rho_0[1 - \alpha(T - T_0)]$, and the relations $T_* = T_*(z)$, $P_* = P_*(z)$. Here α is the specific coefficient of thermal expansion, P_* is the pressure far from the vortex axis, and ρ_0, T_0 are the density and temperature at $z = 0$, $r \to \infty$. As a result, the equations of continuity, motion, and energy assume the following form in a cylindrical coordinate system:

$$\partial Q/\partial r = \partial(wr)/\partial z; \tag{2.3}$$

$$u\frac{\partial (vr)}{\partial r} + w\frac{\partial (vr)}{\partial z} = \nu r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (vr)}{\partial r}\right), \tag{2.4}$$

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{\partial p}{\partial r} + \nu\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (ur)}{\partial r}\right),$$

$$-\frac{\partial (wQ)}{\partial r} + \frac{\partial (rw^2)}{\partial z} = -r\frac{\partial p}{\partial z} + \alpha g r\theta + \nu\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right),$$

$$-\frac{\partial (\theta Q)}{\partial r} + \frac{\partial (rw\theta)}{\partial z} = \beta r w + \varkappa\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right),$$

where $Q = -ur$; ν, kinematic viscosity of the liquid; $\varkappa$, thermal diffusivity; g, acceleration of gravity; $\beta = -\partial T_*/\partial z$; $p = (P - P_*)/\rho_0$; and P, pressure. From (2.3) and the first expression (2.4) we obtain an expression for the radius of the core. We determine r_0 from the maximum of the azimuthal velocity. Then, on the basis of c), r_0 is independent of z in the first approximation. To do so, we assess the possibility of neglecting the second term on its left-hand side. We assume for this assessment that the rotation inside the core is of the rigid-body kind. Estimating u from the equation of continuity, we obtain

$$u\partial (vr)/\partial r \sim \Delta W\Gamma_0 r^2/(r_0^2 l), \tag{2.5}$$

$$w\partial (vr)/\partial z \sim W\Delta\Gamma_0 r^2/(r_0^2 l),$$

where Γ_0 is the value of (vr) at the boundary of the core and ΔW, $\Delta\Gamma_0$ are the variations of the quantities over the height l. According to (2.5), it is impossible to neglect the given term if $\Delta W\Gamma_0 \gg W\Delta\Gamma_0$. The latter statement is true because, according to (2.1), $\Gamma_0 \gg \Delta\Gamma_0$ and, as will be shown presently, $\Delta W \sim W$.

We reject the indicated term and make use of assumption c). We obtain the first-approximation equation

$$\frac{d^2A}{d\eta^2} + \frac{Q}{2\nu\eta}\frac{dA}{d\eta} = 0, \quad \eta = r^2/r_0^2. \tag{2.6}$$

We apply the boundary conditions $A(0) = 0$, $A(\infty) = A_\infty$. We use Eqs. (2.3) and (2.6) to determine the structure of Q, arriving at an expression for r_0. It follows from (2.6) that $Q = Q(r)$ in the first approximation. Then, according to (2.3), w is a linear function of z:

$$w \sim z. \tag{2.7}$$

Solving (2.3) with regard for (2.2) and (2.7), we obtain

$$Q = \begin{cases} Q_0\eta, & 0 \leqslant \eta \leqslant 1, \\ Q_0 = r_0^2\Delta W/(2l), & \eta \geqslant 1. \end{cases} \tag{2.8}$$

By definition, the azimuthal velocity is a maximum at the point r_0. We integrate (2.6) with regard for (2.8). We analyze the resulting expression for an extremum of the azimuthal velocity. We deduce as a result that Q_0 satisfies the equation $\exp(Q_0/(2\nu)) - 1 = Q_0/\nu$. Hence, $Q_0 \approx 2\nu$. We put $Q_0 = 2\nu$ in the first approximation. Then from (2.8) we find the required expression for r_0:

$$r_0^2 = 4\nu l/(\Delta W). \tag{2.9}$$

Consequently, to ascertain the dependence of r_0 on the parameters of the model we must determine ΔW. We find the latter from the equation of motion for the axial component of the velocity and the energy equation. We solve them, making use of assumptions d) and f). We introduce the dimensionless variables

$$W_0 = W/\sqrt{\alpha\beta g l},\ \Theta_0 = \Theta/\beta l,\ \xi = z/l. \tag{2.10}$$

We integrate the equation for w with respect to r from 0 to r_0 and with respect to z from 0 to l and integrate the energy equation with respect to r from 0 to r_0. We substitute (2.10) into the resulting expressions and interpret β as its height-average value.

We obtain

$$W_0^2(1) - W_0^2(0) = -\Delta p_0 + \int_0^1 \Theta_0 d\xi + \frac{2\nu}{r_0\alpha\beta g l}\int_0^1 \left(\frac{\partial w}{\partial r}\right)_0 d\xi, \tag{2.11}$$

$$\frac{d(W_0\Theta_0)}{d\xi} = W_0 + \frac{2\varkappa}{r_0\beta\sqrt{\alpha\beta g}\, l}\left(\frac{\partial\theta}{\partial r}\right)_0;$$

$$\Delta p_0 = \frac{2}{r_0^2\alpha\beta g l^2}\int_0^{r_0}(p(1) - p(0))\, r dr. \tag{2.12}$$

The subscript 0 signifies that the derivatives are evaluated at $r = r_0$. We obtain order-of-magnitude estimates of the derivatives

$$(\partial w/\partial r)_0 \approx -W/r_0,\ (\partial\theta/\partial r)_0 \approx -\Theta/r_0. \tag{2.13}$$

On the basis of (2.7) we seek a solution of (2.11) in the form of linear functions with respect to ξ:

$$W_0 = a_1\xi + b_1,\ \Theta_0 = a_2\xi + b_2. \tag{2.14}$$

The quantities a_1, b_1, a_2, b_2 do not depend on ξ. Their physical significance follows directly from (2.10) and (2.14). We substitute (2.10) and (2.14) into (2.9). We obtain

$$r_0^2 = 4\nu/a_1\sqrt{\alpha\beta g}. \tag{2.15}$$

Substituting (2.13) and (2.14) into (2.11) and replacing r_0^2 by its value (2.15), we have

$$5a_1^2/4 + 5a_1b_1/2 = -\Delta p_0 + a_2/2 + b_2,$$

$$a_2 = \frac{1}{2}\left(1 + \frac{1}{4\,\mathrm{Pr}}\right)^{-1},\quad b_1 = 2a_1b_2\left(1 + \frac{1}{4\,\mathrm{Pr}}\right),$$

where $\mathrm{Pr} = \nu/\varkappa$. Since $4\mathrm{Pr} \gg 1$, we obtain in the first approximation

$$a_1 = \sqrt{\frac{1}{5}\left(1 - \frac{4\Delta p_0}{1 + 4b_2}\right)}. \tag{2.16}$$

Thus, for solutions of the form (2.14) only one out of the four variables a_1, b_1, a_2, b_2 is specified independently, while the others are determined by solving the system (2.11). It is physically clear that b_1 or b_2, i.e., the value of the velocity or temperature at the lower boundary (for ξ = 0) can be specified. However, it is difficult to assign b_1, because in the vortex explosion process there can occur an unmonitorable entrainment of liquid from the exterior region into the core or, conversely, expulsion from the core [9]. We therefore adopt b_2 as the free parameter. We infer from (2.16) and (2.15) that to find the value of r_0 it is necessary to specify b_2 and β and to determine Δp_0. The heating conditions were constant (a dn fixed) in the present experiment, and so it may be assumed that b_2 and β are approximately constant. Then the variations of r_0 are completely determined by the variations of Δp_0.

We determine Δp_0 and, in order to explain the core compaction effect, analyze the dependence of Δp_0 on A_∞ and l. The equation of motion for the radial component of the velocity in the first approximation has the form

$$\partial p/\partial r = A^2/r^3. \tag{2.17}$$

Assuming that the rotation is of the rigid-body type inside the core and is potential outside it, by means of (2.17) we obtain

$$\int_0^{r_0} p(1)\, r dr = -3A_\infty^2/8. \tag{2.18}$$

For the determination of Δp_0 it is required to find the difference between (2.18) and the integral

$$\int_0^{r_0} p(0)\, r dr.$$

We determine the latter, taking into account the conditions at the lower boundary, namely the vortex explosion effect. We consider the region AA'B'B (Fig. 5). We denote by r_1 the radius of the vortex before explosion, and by u_1 the average velocity in the cross section CC'. We write the integral law of conservation of the axial component of the momentum for AA'B'B:

$$\int_0^{r_0} p(0)\, r dr = -\frac{3}{8} A_\infty^2 - \frac{A_\infty^2}{2} \ln \frac{r_0}{r_1} + \frac{u_1^2 r_1^2}{2}. \tag{2.19}$$

The following assumptions are basic to the derivation of (2.19). Outside the region CDEAA' E'D'C' the liquid has a negligibly small vertical velocity. The segment CD is sufficiently short to permit neglect of friction and the buoyant force acting on the segment. Inside CC' the rotation is of the rigid-body type, and outside the segment it is potential. Equation (2.17) is valid in the region BEE'B' as well. In addition, on the basis of the experimental data, in (2.19) we reject terms corresponding to the momentum flux in the cross section AA' as well as the buoyant force and the friction force acting on AA'E'E. To estimate them we use tabulated data, along with the quantities W(0) = 1.6 cm/sec and AE = 1 cm. We obtain an expression for $u_1^2 r_1^2$. The quantity $u_1 r_1^2$ is equal to the mass flux in the boundary layer, divided by π. For laminar flow Rott and Lewellen [16] give the estimate $u_1 r_1^2 \sim (\nu A_\infty)^{1/2} \cdot$ R, where R is the order of the radius of the cylinder. It has been shown [17] that the diameter of the vortex before explosion is equal in order of magnitude to the boundary-layer thickness δ. This fact has also been noted in [9]. According to [16], $\delta \sim (\nu/A_\infty)^{1/2}$ R. Then, $u_1^2 r_1^2 = kA_\infty^2$, where k is a certain constant coefficient. Using the latter expression, we substitute (2.18) and (2.19) into (2.12). We obtain

$$\Delta p_0 = \frac{A_\infty^2}{\alpha\beta g l^2 r_0^2}\left(\ln \frac{r_0}{r_1} - k\right). \tag{2.20}$$

We show that if the expression in the parentheses is positive, then r_0 will be an increasing function of A_∞ and a decreasing function of l. We assume the opposite. Then Δp_0 increases with increasing A_∞ or decreasing l. The latter is true, since according to experiment, $r_0/r_1 \gg 10$, and, according to [16], r_1 is a decreasing function of A_∞. The growth of Δp_0 causes r_0 to increase, contradicting the original assumption.

We substitute (2.20) and (2.16) into (2.15). We solve the resulting expression for r_0^2. We find

$$r_0^2 = \frac{4\sqrt{5}\nu}{\sqrt{\alpha\beta g}}\left(\sqrt{c^2+1} + c\right), \tag{2.21}$$

where

$$c=\frac{(\ln(r_0/r_1)-k)A_\infty^2}{2\sqrt{5}(1+4b_2)\sqrt{\alpha\beta g}\,l^2\nu}.$$

Substituting (2.21) for (2.15), we get

$$a_1=\frac{1}{\sqrt{5}(\sqrt{c^2+1}+c)}. \tag{2.22}$$

We can use (2.10), (2.14), and (2.22) to determine W(0) and ΔW.

We have thus derived expressions for estimates of the vortex parameters. It follows from their derivation that, according to the model, the boundary layer at the bottom surface exerts an appreciable influence on the external flow. Vortex explosion is the principal factor in this influence.

We note a departure of the models in [12, 14] from the one discussed here. In [12] the flow is assumed to be turbulent, and the liquid inside the core is entrained due to turbulent mixing processes at its boundary. The continuity equation obtained in this case does not have solutions in which $W \sim z$ and r_0 is independent of z. Conversely, according to [12], r_0 increases with the height z. The model proposed in [14] is closer to the present one. It is deduced in [14] that r_0 does not depend on z. For the given stratification of the medium r_0 depends only on the viscosity, $r_0 \sim \nu^{1/2}$. The existence of the boundary layer at the bottom surface is practically ignored in [14]. We note that expression (2.21) for $c \to 0$ coincides with the expression for r_0^2 in [14], correct to within a coefficient of order unity. In both [12] and [14] the height of the vortex does not enter into the parameters of the problem.

3. We now compare the given model with the experimental.

1. The core compaction effect with increasing A_∞ or decreasing l is explained within the context of the model if the heating conditions are fixed. Since r_0 increases with increasing A_∞ or with decreasing l, the quantity $A_\infty/(r_0 l)$ must vary less appreciably than each of the quantities A_∞, r_0, l taken separately. This result is consistent with experiment (see Table 1).

We investigate the possibility of using expressions (2.21) and (2.22) to calculate the parameters of vortices. According to (2.21) and (2.22), the only unknown is $\ln(r_0/r_1) - k$. Inasmuch as r_0/r_1 did not vary more than an order of magnitude in the experiment and the minimum value of r_0/r_1 is of the order of 10, we can regard the quantity $\ln(r_0/r_1) - k$ as constant in the first approximation. To evaluate it we make use of the fact that r_0 is practically invariant for $f \leqslant 0.9$ rps and $l = 12$ cm. The latter result implies that the data of the first column in Table 1 corresponds to small c. For better correspondence with the experiment we assume for the first column that c = 0.2.

2. We determine the theoretical values of W(0), ΔW, and r_0 for c = 0.2. We set $\rho_0 = 1$ g/cm^3, $\alpha = 2\cdot10^{-4}$ (°C)$^{-1}$, $\nu = 0.7\cdot10^{-2}$ cm^2/sec, $\beta = 0.1$ °C/cm, and $b_2 = 1$. Then from (2.10), (2.14), (2.21), and (2.22) we obtain $W(0) \approx 1.4$ cm/sec, $\Delta W \approx 0.7$ cm/sec, and $r_0 \approx 0.7$ cm.

The corresponding experimental values are $W(0) \approx 1.6$ cm/sec, $\Delta W \approx 0.6$ cm/sec, and $r_0 \approx 0.3$ cm (here W(0) is the maximum value of w at a height of 1 cm, ΔW is calculated from the difference between the w maxima at heights of 1 and 3 cm, and r_0 is taken from the first column of the table).

3. Using expression (2.22), we calculate the ratios of the core radii corresponding to the values of A_∞ and l from columns 2-4 of the table to the value of r_0 corresponding to A_∞ and l in column 1, and of the radius r_0 corresponding to A_∞ and l from column 3 to r_0 from columns 2 and 4. We obtain 1.3, 2.4, 1.3, 1.8, and 1.8.

The analogous ratios for the radii r_0 taken from the table are equal to 1.8, 3.7, 1.8, 2, and 2.

Thus, the theoretical results are qualitatively consistent with the experimental, and the calculated values of the vortex parameters according to the model equations agree in order of magnitude with their experimental values.

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